Chapter 6 Problem Set – Trigonometric Identities and Equations

6.1 Basic Trigonometric Functions #3cdef, 5cdef (pg 392 in textbook)

3. Use the cofunction identities to write an expression that is equivalent to each of the following expressions.

c) $\tan\frac{3\pi}{8}$ e) $\sin\frac{\pi}{8}$

d) $\cos\frac{5\pi}{16}$ f) $\tan\frac{\pi}{6}$

5. Write an expression that is equivalent to each of the following expressions, using the related acute angle.

c) $\tan\frac{5\pi}{4}$ e) $\sin\frac{13\pi}{8}$

d) $\cos\frac{11\pi}{6}$ f) $\tan\frac{5\pi}{3}$

6.2 Compound Angle Formulae #3 – 6, 8 – 10, 13 (pg 400 in textbook)

3. Express each angle as a compound angle, using a pair of angles from the special triangles.

a) $75°$ c) $-\frac{\pi}{6}$ e) $105°$

b) $-15°$ d) $\frac{\pi}{12}$ f) $\frac{5\pi}{6}$

4. Determine the exact value of each trigonometric ratio.

a) $\sin 75°$ c) $\tan\frac{5\pi}{12}$ e) $\cos 105°$

b) $\cos 15°$ d) $\sin\left(-\frac{\pi}{12}\right)$ f) $\tan\frac{23\pi}{12}$

5. K Use the appropriate compound angle formula to determine the exact value of each expression.

a) $\sin\left(\pi+\frac{\pi}{6}\right)$ c) $\tan\left(\frac{\pi}{4}+\pi\right)$ e) $\tan\left(\frac{\pi}{3}-\frac{\pi}{6}\right)$

b) $\cos\left(\pi-\frac{\pi}{4}\right)$ d) $\sin\left(-\frac{\pi}{2}+\frac{\pi}{3}\right)$ f) $\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$

6. Use the appropriate compound angle formula to create an equivalent expression.

a) $\sin(\pi+x)$ c) $\cos\left(x+\frac{\pi}{2}\right)$ e) $\sin(x-\pi)$

b) $\cos\left(x+\frac{3\pi}{2}\right)$ d) $\tan(x+\pi)$ f) $\tan(2\pi-x)$

8. Determine the exact value of each trigonometric ratio.

a) $\cos 75°$ c) $\cos \frac{11\pi}{12}$ e) $\tan \frac{7\pi}{12}$

b) $\tan (-15°)$ d) $\sin \frac{13\pi}{12}$ f) $\tan \frac{-5\pi}{12}$

9. If $\sin x = \frac{4}{5}$ and $\sin y = -\frac{12}{13}$, $0 < x < \frac{\pi}{2}$, $\frac{3\pi}{2} < y < 2\pi$, evaluate

a) $\cos (x + y)$ c) $\cos (x - y)$ e) $\tan (x + y)$

b) $\sin (x + y)$ d) $\sin (x - y)$ f) $\tan (x - y)$

10. A α and β are acute angles in quadrant I, with $\sin \alpha = \frac{7}{25}$ and $\cos \beta = \frac{5}{13}$. Without using a calculator, determine the values of $\sin (\alpha + \beta)$ and $\tan (\alpha + \beta)$.

13. T Simplify $\frac{\sin (f + g) + \sin (f - g)}{\cos (f + g) + \cos (f - g)}$.

6.3 Double Angle Formulae #2, 4, 6, 7, 12 (pg 407 in textbook)

2. Express each of the following as a single trigonometric ratio and then evaluate.

b) $\cos^2 30° - \sin^2 30°$ d) $\cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12}$

c) $2 \sin \frac{\pi}{12} \cos \frac{\pi}{12}$ f) $2 \tan 60° \cos^2 60°$

4. K Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos \theta = \frac{3}{5}$ and $0 \leq \theta \leq \frac{\pi}{2}$.

6. Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\sin \theta = -\frac{12}{13}$ and $\frac{3\pi}{2} \leq \theta \leq 2\pi$.

7. Determine the values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$, given $\cos \theta = -\frac{4}{5}$ and $\frac{\pi}{2} \leq \theta \leq \pi$.

12. Use the appropriate compound angle formula and double angle formula to develop a formula for

a) $\sin 3\theta$ in terms of $\cos \theta$ and $\sin \theta$

b) $\cos 3\theta$ in terms of $\cos \theta$ and $\sin \theta$

c) $\tan 3\theta$ in terms of $\tan \theta$

6.4 Trigonometric Identities #8 – 11 (pg 417 in textbook)

8. Prove that $\frac{1 + \tan x}{1 + \cot x} = \frac{1 - \tan x}{\cot x - 1}$.

9. Prove each identity.

a) $\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta} = 1 - \tan\theta$

b) $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$

c) $\tan^2 x - \cos^2 x = \frac{1}{\cos^2 x} - 1 - \cos^2 x$

d) $\frac{1}{1 + \cos\theta} + \frac{1}{1 - \cos\theta} = \frac{2}{\sin^2\theta}$

10. Prove each identity.

a) $\cos x \tan^3 x = \sin x \tan^2 x$

b) $\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$

c) $(\sin x + \cos x)\left(\frac{\tan^2 x + 1}{\tan x}\right) = \frac{1}{\cos x} + \frac{1}{\sin x}$

d) $\tan^2\beta + \cos^2\beta + \sin^2\beta = \frac{1}{\cos^2\beta}$

e) $\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos x$

f) $\sin\left(\frac{\pi}{2} - x\right)\cot\left(\frac{\pi}{2} + x\right) = -\sin x$

11. Prove each identity.

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a) $\frac{\cos 2x + 1}{\sin 2x} = \cot x$

b) $\frac{\sin 2x}{1 - \cos 2x} = \cot x$

c) $(\sin x + \cos x)^2 = 1 + \sin 2x$

d) $\cos^4\theta - \sin^4\theta = \cos 2\theta$

e) $\cot\theta - \tan\theta = 2\cot 2\theta$

f) $\cot\theta + \tan\theta = 2\csc 2\theta$

g) $\frac{1 + \tan x}{1 - \tan x} = \tan\left(x + \frac{\pi}{4}\right)$

h) $\csc 2x + \cot 2x = \cot x$

i) $\frac{2\tan x}{1 + \tan^2 x} = \sin 2x$

j) $\sec 2t = \frac{\csc t}{\csc t - 2\sin t}$

k) $\csc 2\theta = \frac{1}{2}(\sec\theta)(\csc\theta)$

l) $\sec t = \frac{\sin 2t}{\sin t} - \frac{\cos 2t}{\cos t}$

6.5 Linear Trigonometric Equations #6, 7def, 8, 9abc (pg 427 in textbook)

6. Determine the solutions for each equation, where $0 \le \theta \le 2\pi$.

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a) $\tan\theta = 1$

b) $\sin\theta = \frac{1}{\sqrt{2}}$

c) $\cos\theta = \frac{\sqrt{3}}{2}$

d) $\sin\theta = -\frac{\sqrt{3}}{2}$

e) $\cos\theta = -\frac{1}{\sqrt{2}}$

f) $\tan\theta = \sqrt{3}$

7. Using a calculator, determine the solutions for each equation on the interval $0° \le \theta \le 360°$. Express your answers to one decimal place.

d) $-3\sin\theta - 1 = 1$

e) $-5\cos\theta + 3 = 2$

f) $8 - \tan\theta = 10$

8. Using a calculator, determine the solutions for each equation, to two decimal places, on the interval $0 \le x \le 2\pi$.

a) $3\sin x = \sin x + 1$

b) $5\cos x - \sqrt{3} = 3\cos x$

c) $\cos x - 1 = -\cos x$

d) $5\sin x + 1 = 3\sin x$

9. Using a calculator, determine the solutions for each equation, to two decimal places, on the interval $0 \le x \le 2\pi$.

a) $2 - 2\cot x = 0$

b) $\csc x - 2 = 0$

c) $7\sec x = 7$

6.5 Quadratic Trigonometric Equations #4ade, 5acef, 6ac, 7 - 9 (pg 436 in textbook)

4. Solve for θ, to the nearest degree, in the interval $0° \le \theta \le 360°$.

a) $\sin^2\theta = 1$

d) $4\cos^2\theta = 1$

e) $3\tan^2\theta = 1$

5. Solve each equation for x, where $0° \le x \le 360°$.

a) $\sin x \cos x = 0$

c) $(\sin x + 1)\cos x = 0$

e) $(\sqrt{2}\sin x - 1)(\sqrt{2}\sin x + 1) = 0$

f) $(\sin x - 1)(\cos x + 1) = 0$

6. Solve each equation for x, where $0 \le x \le 2\pi$.

a) $(2\sin x - 1)\cos x = 0$

c) $(2\cos x + \sqrt{3})\sin x = 0$

7. Solve for θ to the nearest hundredth, where $0 \leq \theta \leq 2\pi$.

a) $2\cos^2\theta + \cos\theta - 1 = 0$

b) $2\sin^2\theta = 1 - \sin\theta$

c) $\cos^2\theta = 2 + \cos\theta$

d) $2\sin^2\theta + 5\sin\theta - 3 = 0$

e) $3\tan^2\theta - 2\tan\theta = 1$

f) $12\sin^2\theta + \sin\theta - 6 = 0$

8. Solve each equation for x, where $0 \leq x \leq 2\pi$.

a) $\sec x \csc x - 2\csc x = 0$

b) $3\sec^2 x - 4 = 0$

c) $2\sin x \sec x - 2\sqrt{3}\sin x = 0$

d) $2\cot x + \sec^2 x = 0$

e) $\cot x \csc^2 x = 2\cot x$

f) $3\tan^3 x - \tan x = 0$

9. Solve each equation in the interval $0 \leq x \leq 2\pi$. Round to two decimal places, if necessary.

a) $5\cos 2x - \cos x + 3 = 0$

b) $10\cos 2x - 8\cos x + 1 = 0$

c) $4\cos 2x + 10\sin x - 7 = 0$

d) $-2\cos 2x = 2\sin x$